Code No: R10206/R10

## Set No. 1

I B.Tech II Semester Supplementary Examinations, Feb/Mar 2014 MATHEMATICAL METHODS
( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find rank of a Matrix using Echelon form where $A=\left[\begin{array}{cccc}1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4\end{array}\right]$
(b) Show that equations $\mathrm{x}+\mathrm{y}+\mathrm{z}=6, \mathrm{x}+2 \mathrm{y}+3 \mathrm{z}=14 \mathrm{x}+4 \mathrm{y}+7 \mathrm{z}=30$ are consistent and solve them
$[7+8]$
2. Verify Cayley - Hamilton theorem, find $\mathrm{A}^{-1}$ and $\mathrm{A}^{3}$ if $A=\left[\begin{array}{ccc}3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5\end{array}\right]$
3. Reduce the quadratic form $3 x^{2}+5 y^{2}+3 z^{2}-2 x y-2 y z+2 z x$ to the canonical form by orthogonal reduction. Also find its nature and Signature
[15]
4. (a) Find the root of the equation $x^{3}-6 x+4=0$ by Newton-Raphson's Method correct to five decimal places
(b) Find a root of the equation $x \log _{10} x=1.2$ by Bisection method.
5. (a) Find $y(1.6)$ using Newton's forward difference formula from the table

| X | 1 | 1.4 | 1.8 | 2.2 |
| :--- | :--- | :--- | :--- | :--- |
| Y | 3.49 | 4.82 | 5.96 | 6.5 |

(b) Using Gauss's forward interpolation formula find y at $\mathrm{x}=1.7489$ given that

| $\mathrm{X}:$ | 1.72 | 1.73 | 1.74 | 1.75 | 1.76 | 1.77 | 1.78 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 0.1791 | 0.1773 | 0.1775 | 0.1738 | 0.1720 | 0.1703 | 0.1686 |$\quad[8+7]$

6. (a) The population of a certain town (as obtained from census data) is shown in the following table:

| Year | 1891 | 1901 | 1911 | 1921 | 1931 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Population(in <br> thousand) | 46 | 66 | 81 | 93 | 101 |

Estimate the rate of growth of the population in the year 1921
(b) When a train is moving at $30 \mathrm{~m} / \mathrm{sec}$, steam is shut off and brakes are applied.

The speed of the train per second after t seconds is given by

| Time (t): | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Speed (v): | 30 | 24 | 19.5 | 16 | 13.6 | 11.7 | 10 | 8.5 | 7.0 |

Using Simpson's rule, determine the distance moved by the train in 40 seconds. [8+7]
7. (a) Solve $y^{1}=1+x y$ subject to the condition $y(0)=1$ by Taylor series method and hence find $y(0.2)$
(b) Solve $y^{1}=x+y+x y$ subject to the condition $y(0)=1$ by Picard's method and hence find $y(0.1)$
8. (a) Fit a least square straight line to the following data

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 16 | 19 | 23 | 26 | 30 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.1 | 3.5 | 5.4 | 7.3 | 8.2 |

Code No: R10206/R10

## Set No. 2

I B.Tech II Semester Supplementary Examinations, Feb/Mar 2014 MATHEMATICAL METHODS
( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find rank of matrix $A=\left[\begin{array}{ccc}1 & 2 & 3 \\ 2 & -2 & 0 \\ 3 & 1 & 4 \\ -2 & 3 & 1\end{array}\right]$ using Normal form.
(b) Solve system of equations, if consistent $x+y+2 z=4,2 x-y+3 z=9,3 x-y-z=2$
2. Using Cayley - Hamilton theorem find $\mathrm{A}^{8}$ if $A=\left[\begin{array}{cc}1 & 2 \\ 2 & -1\end{array}\right]$
3. Find the transformation which will transform $4 x^{2}+3 y^{2}+z^{2}-8 x y-6 y z+4 z x$ into a sum of square and find the reduced from
4. (a) Find a real root of the equation $x^{3}-x-4=0$, using Regula - Falsi method.
(b) Find a real root of the equation $x e^{x}-\cos x=0$ using Newton-Raphson's method.
5. (a) Using the Gauss forward interpolation formula, find the value of $\log _{10} 347.5$ from the following table

| x | 320 | 330 | 340 | 350 | 360 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\log _{10}$ | 2.5052 | 2.5185 | 2.5315 | 2.5441 | 2.5563 |

(b) Compute the approximate value of $e^{0.35}$, using the following table:

| x | 0 | 0.1 | 0.2 | 0.3 | 0.4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $e^{x}$ | 1.0000 | 1.1052 | 1.2214 | 1.3499 | 1.4918 |

6. (a) The population of a certain town (as obtained from census data) is shown in the following table:

| Year | 1951 | 1961 | 1971 | 1981 | 1991 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Population(in <br> thousand) | 19.96 | 39.65 | 58.81 | 77.21 | 94.61 |

Estimate the rate of growth of the population in the year 1981
(b) The following table gives the value of $\mathrm{f}(\mathrm{x})$ at equal intervals of x .

| X | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 0.399 | 0.352 | 0.242 | 0.129 | 0.054 |
|  |  |  |  |  |  |

Evaluate $\int_{0}^{2} f(x) d x$ using Simpsons $1 / 3$ and Simpsons $3 / 8$ rule.
7. (a) Solve $y^{1}=1-y$ subject to the condition $y(0)=0$ by Taylor series method hence find $\mathrm{y}(0.1), \mathrm{y}(0.2)$
(b) Solve $\mathrm{y}^{1}=1+\mathrm{y}^{2}$ subject to the condition $\mathrm{y}(0)=0$ by Picard's method $[8+7]$
8. (a) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 2 | 3 | 5 | 8 | 10 |

(b) Find the best fit of the type $\mathrm{y}=\mathrm{ae}^{b x}$ to the data by the method of least squares

| x | 1 | 1.2 | 1.4 | 1.6 |
| :--- | :--- | :--- | :--- | :--- |
| y | 40.17 | 73.196 | 133.372 | 243.02 |

$$
[8+7]
$$

Code No: R10206/R10

## Set No. 3

I B.Tech II Semester Supplementary Examinations, Feb/Mar 2014 MATHEMATICAL METHODS
( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find rank of matrix using Echelon form $A=\left[\begin{array}{ccc}1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3\end{array}\right]$
(b) Solve the equations using Gauss Jordan method
$\mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3}=8,2 \mathrm{x}_{1}+3 \mathrm{x}_{2}+2 \mathrm{x}_{3}=19,4 \mathrm{x}_{1}+2 \mathrm{x}_{2}+3 \mathrm{x}_{3}=23$
2. (a) Find Eigen Vectors of $A=\left[\begin{array}{ccc}-2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0\end{array}\right]$
(b) The Eigen vectors corresponding to two different eigen values are linearly independent
3. Reduce the quadratic form $3 x^{2}+5 y^{2}+3 z^{2}-2 x y-2 y z+2 z x$ to the canonical form by orthogonal reduction. Also find its nature and Signature
4. (a) Prove that $\sqrt[b]{a}$ can be evaluated by using the iterative procedure
$x_{n+1}=\frac{1}{b}\left\{(b-1) x_{n}+\frac{a}{x_{n}^{b-1}}\right\}$ and hence find $\sqrt[3]{2}$
(b) Find the real root of the equation $x^{3}-x-1=0$ by Bisection method. [7+8]
5. (a) Prove the following.
(i) $\triangle \nabla=\triangle-\nabla$
(ii) $\triangle E=E \nabla=\nabla$
(b) From the following table of values of $y=f(x)$, find $f(0.53)$, using the Newton's backward interpolation formula.

| x | 0.30 | 0.40 | 0.50 | 0.60 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\mathrm{f}(\mathrm{x})$ | 0.6179 | 0.6554 | 0.6915 | 0.6915 |

6. (a) From the following data find $f^{\prime}(0.5)$

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 4 | 8 | 15 | 7 | 6 | 2 |

(b) Evaluate $\int_{0}^{1} e^{-x^{2}}$ dx taking $\mathrm{h}=0.2$ using
(i) Simpson's $\frac{1}{3} r d$ rule (ii) Trapezoidal rule.
7. (a) Solve $y^{1}=1-y, y(0)=0$ by Euler's method and find $y$ at $x=0.1,0.2$
(b) Solve $\mathrm{y}^{1}=\mathrm{y}-\mathrm{x}, \mathrm{y}(0)=2, \mathrm{~h}=0.2$, by fourth order $\mathrm{R}-\mathrm{K}$ method and hence find $\mathrm{y}(0.2)$
8. (a) Fit a curve of the type $\mathrm{y}=\mathrm{ae}^{b x}$ to the data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 20 | 30 | 52 | 77 | 135 | 211 | 326 | 550 | 1052 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0.0 | 0.2 | 0.4 | 0.7 | 0.9 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1.016 | 0.768 | 0.648 | 0.401 | 0.272 | 0.193 |$\quad[7+8]$

Code No: R10206/R10

## Set No. 4

I B.Tech II Semester Supplementary Examinations, Feb/Mar 2014 MATHEMATICAL METHODS
( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find rank of matrix $A=\left[\begin{array}{ccc}1 & 2 & 3 \\ 2 & -2 & 0 \\ 3 & 1 & 4 \\ -2 & 3 & 1\end{array}\right]$ using Normal form.
(b) Solve system of equations, if consistent $x+y+2 z=4,2 x-y+3 z=9,3 x-y-z=2$
$[7+8]$
2. (a) Prove that the Eigen values of a triangular matrix are diagonal elements of the matrix
(b) Find eigen vectors of $\mathrm{B}=2 \mathrm{~A}^{2}-\mathrm{A}+3 \mathrm{I}$ when $A=\left[\begin{array}{cc}8 & -4 \\ 2 & 2\end{array}\right] \quad[5+10]$
3. Determine the diagonal matrix diagonally similar to the real symmetric matrix $\mathbf{A}=\left[\begin{array}{lll}2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2\end{array}\right]$
4. (a) solve the equation $\mathrm{x}^{3}+2 \mathrm{x}^{2}+0.4=0$ using Newton's -Raphson's Method.
(b) Show that the iteration scheme $\phi(x)=\frac{-1}{x^{2}-3}$ converges and hence find a real root of $f(x)=x^{3}-3 x+1=0$ near $x=3$.
5. (a) (i) Solve $\Delta\left(e^{a x} \log b x\right)$ (ii) Prove that $\nabla^{6} y_{8}=\Delta^{6} y_{2}$.
(b) From the following table for find $f(3.3)$ using gauss forward interpolation formula.

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\mathrm{f}(\mathrm{x})$ | 15.30 | 15.10 | 15.00 | 14.50 | 14.00 |

6. (a) For the following data, find $\frac{d y}{d x} a n d \frac{d^{2} y}{d x^{2}}$ at (i) $\mathrm{x}=1.1$ and (ii) $\mathrm{x}=1.6$

| x | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 7.989 | 8.403 | 8.781 | 9.129 | 9.451 | 9.750 | 10.031 |

(b) Evaluate $\int_{2}^{10} \frac{d x}{1+x}$ using (i) Trapezoidal and (ii) Simpson's $\frac{1}{3}$ rule, taking $\mathrm{h}=$ 1.0 and compare the results with the exact value
7. (a) Solve $y^{1}=x+\sin y, y(0)=1$ by modified Euler's method and hence find $y(o .2)$, $\mathrm{y}(\mathrm{o} .4)$ with $\mathrm{h}=0.2$
(b) Solve $\mathrm{y}^{1}=3 \mathrm{x}-4 \mathrm{y}, \mathrm{y}(0)=2, \mathrm{~h}=0.2$ find $\mathrm{y}(0.4)$ by R-K method
8. (a) Fit a least square straight line to the following data

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 16 | 19 | 23 | 26 | 30 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.1 | 3.5 | 5.4 | 7.3 | 8.2 |

